Nonlinear Fibre Optics

Prof. Luc Thévenaz

Prof. Camille-Sophie Brès

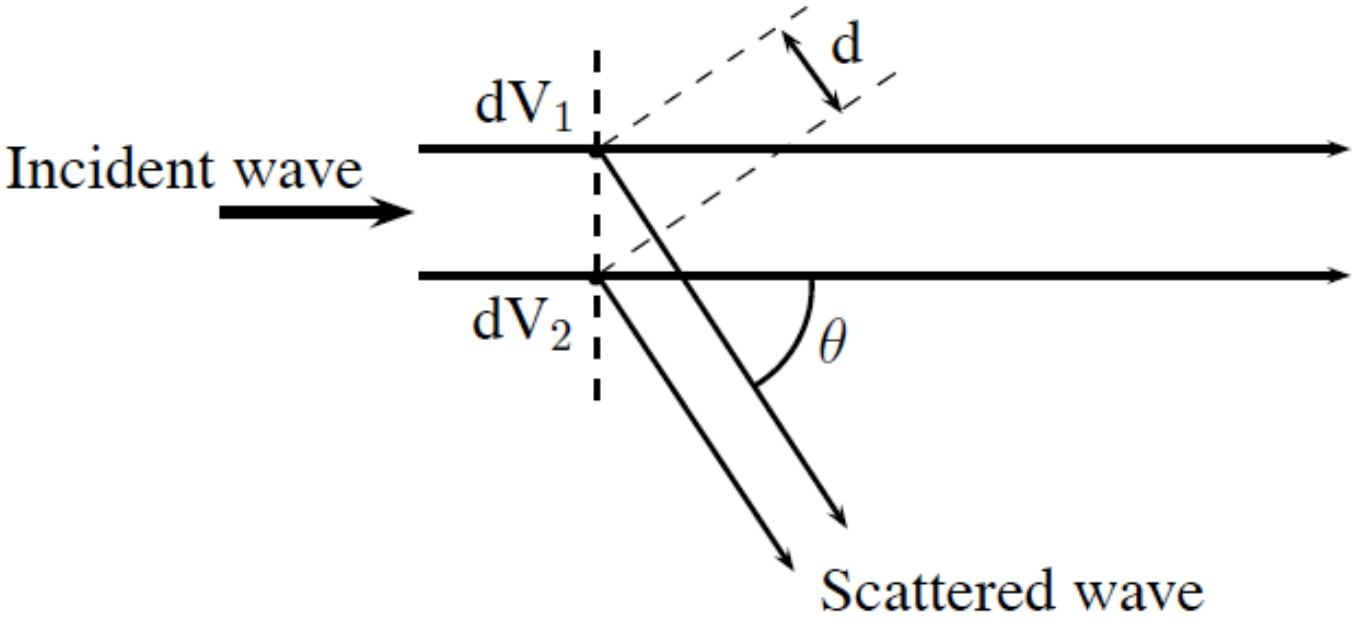
Institute of Electrical Engineering

Doctoral School in Photonics

EPFL

In dense homogeneous ordered media (e.g. crystal), light scatters only in the forward direction

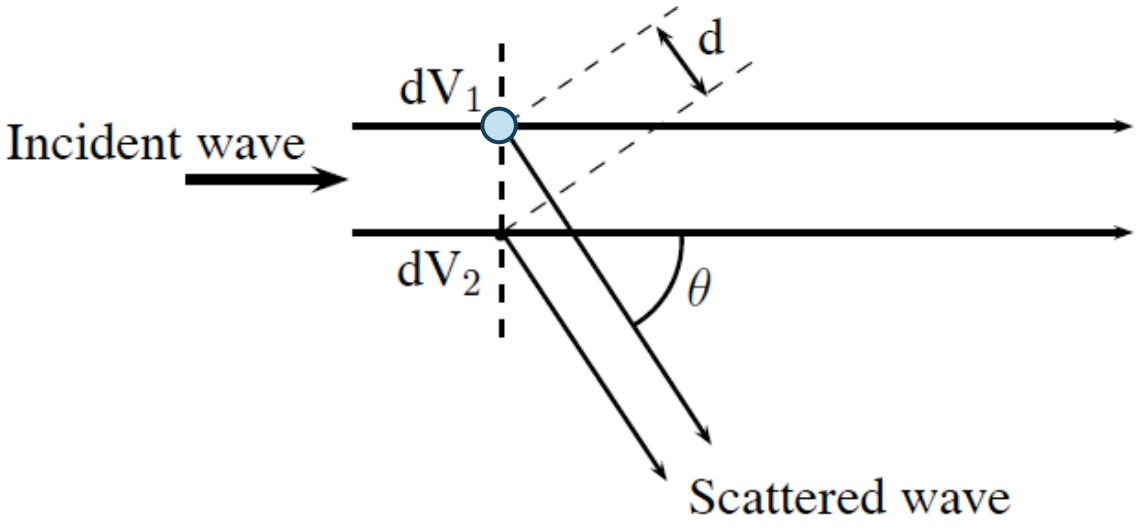
- ***λ* is much larger than the spacing** between scattering centres.
- There always exists an elementary volume dV_2 from which the scattered field will **destructively interferes** with that from dV_1.
- This is **true for all angles *θ* except in the forward direction**.



Credit to Prof. Marc Wuilpart, University of Mons, Belgium

In dense non-homogeneous (disordered) media, light also scatters laterally

- **Density fluctuation** ⇒ the number of scattering centres in volume dV_1 could be different to that of volume from dV_2 ⇒ **the scattered wave in direction θ does not completely vanish**.
- If the local perturbations (local modifications of the optical properties) are separated by distances in the range of λ or more and if they are randomly distributed, the **case is similar to that of the gas**.
- **Any imperfection** that locally modifies the optical properties of the medium (density fluctuation, flaws, impurities) **results in the presence of lateral scattering**.
- **Case of glasses!**



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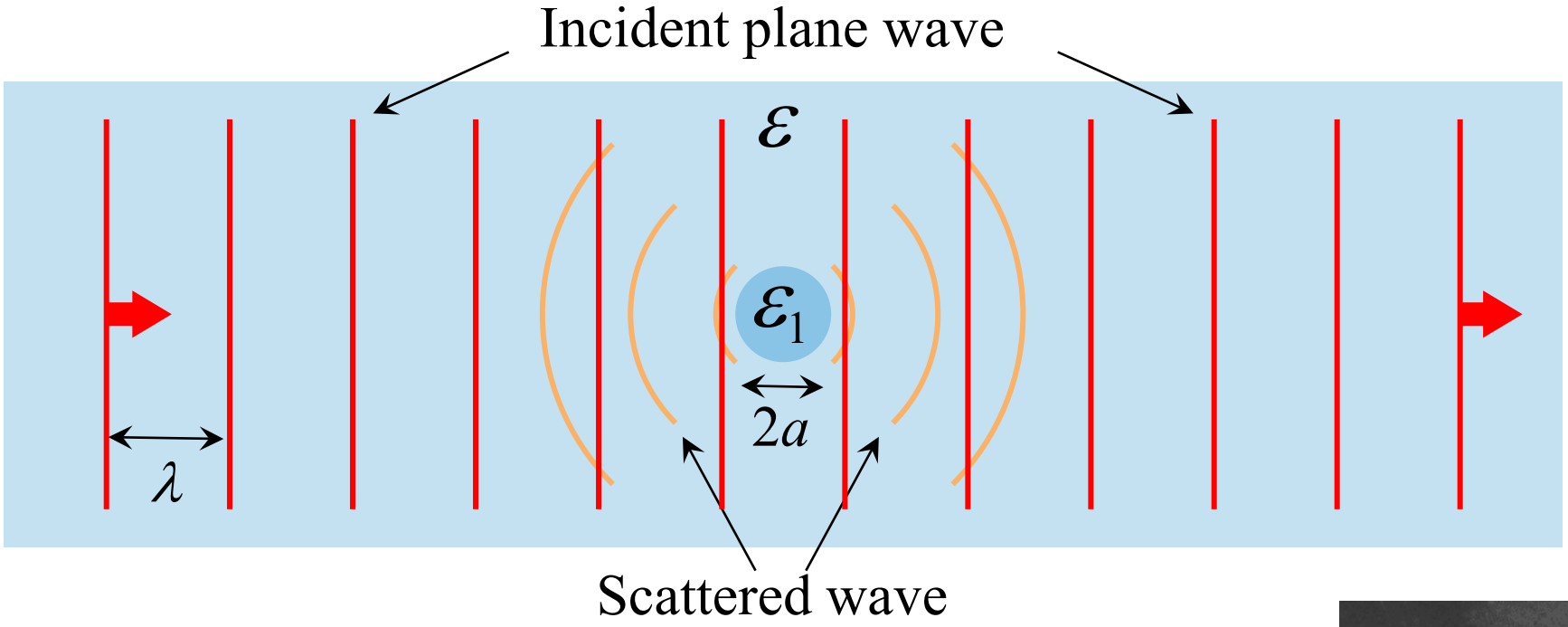
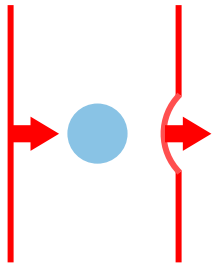
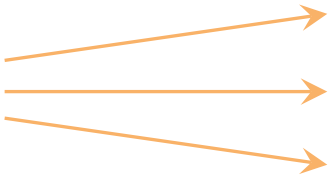
Origin of scattering



First case: $a \geq \lambda$ **Mie scattering**



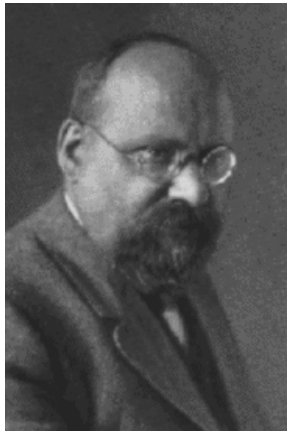


Dominantly forward scattering

Moderate spectral dependence

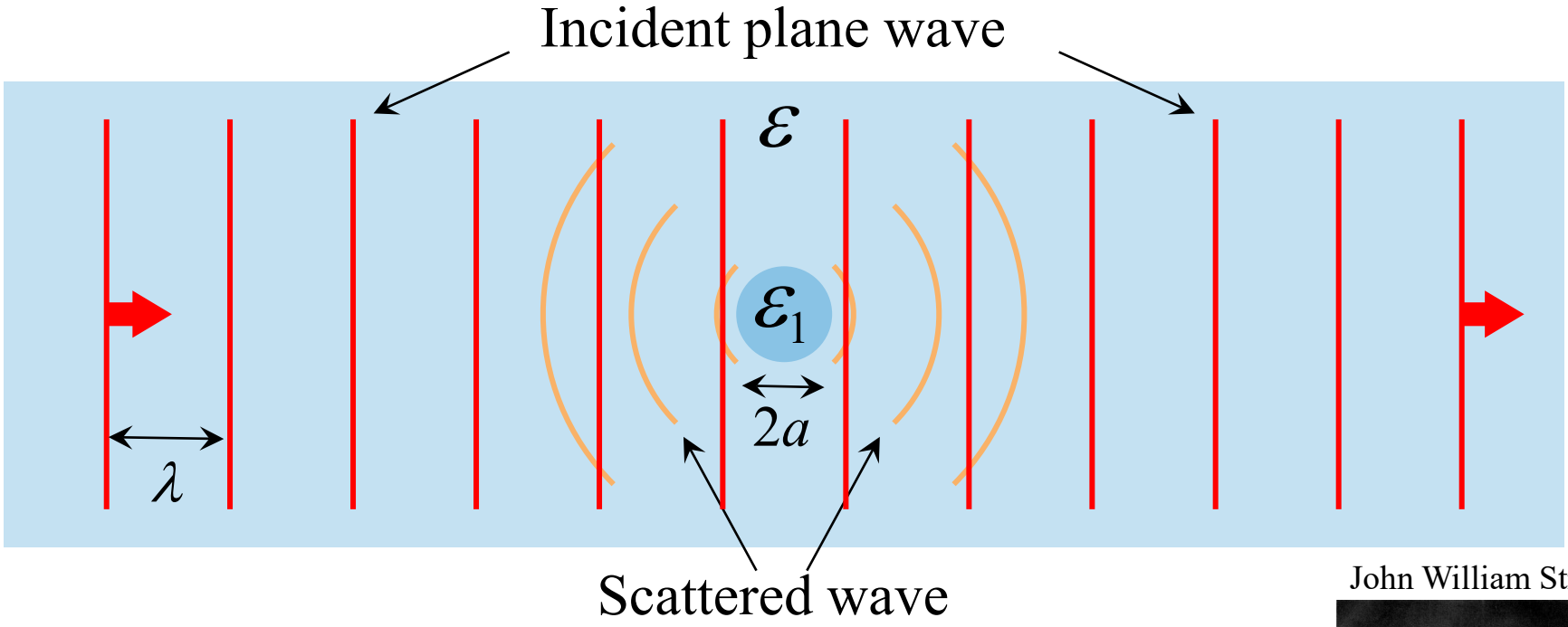
$\sim \lambda$

Diffraction

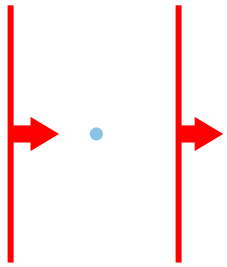


Gustav Mie (1868-1957)

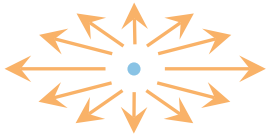
Origin of scattering



Second case: $a << \lambda$ **Rayleigh scattering**



No wavefront distortion

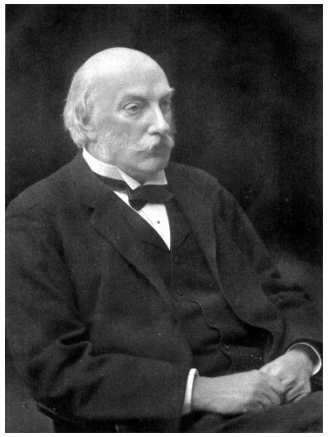


Electromagnetic radiant

All-direction scattering

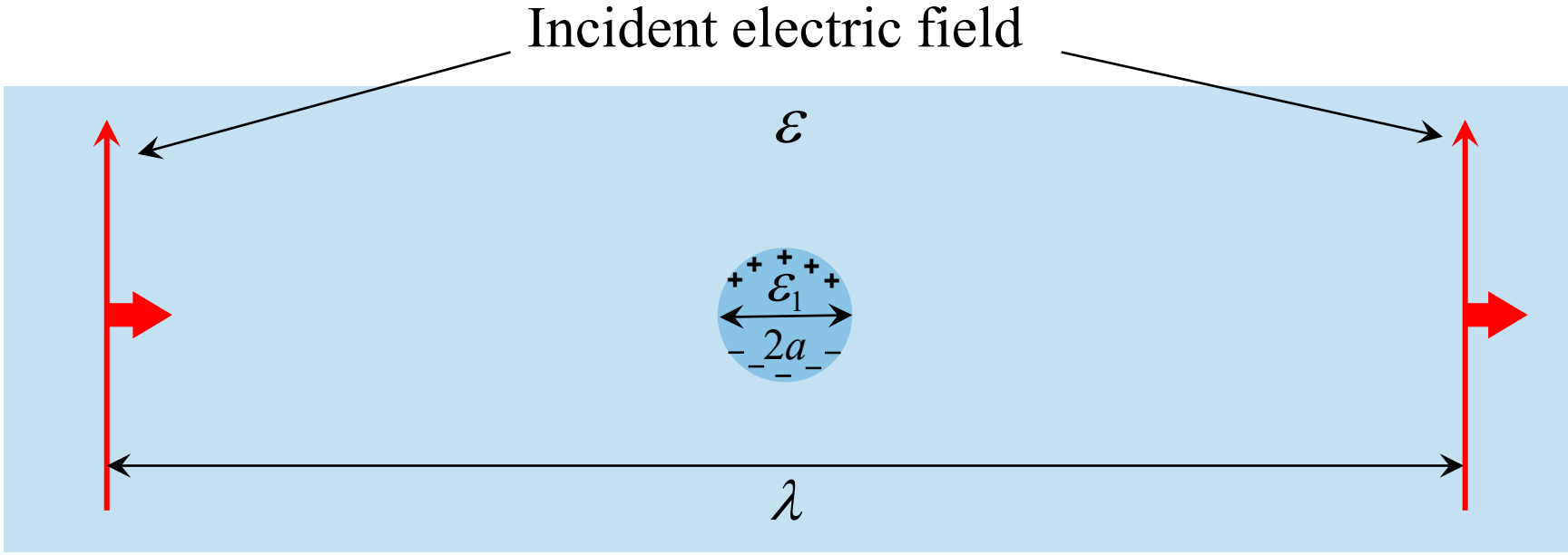
Strong spectral dependence $\sim \lambda^{-4}$

John William Strutt



Baron Rayleigh (1842-1919)

Basic mechanism of Rayleigh scattering

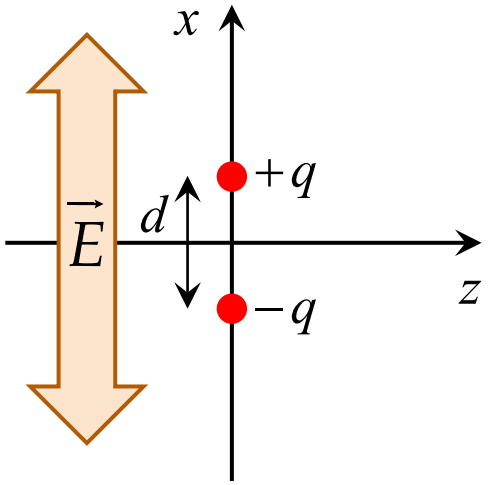


- Since $a << \lambda$ the particle sees a uniform oscillating E-field
- The E-field induces surface charges at the dielectric boundaries of the particle
- Assuming $\varepsilon_1 \cong \varepsilon$ (small medium fluctuation) the surface charges induce a dipole moment

$$\vec{p} = 4\pi\varepsilon\, a^3 \frac{\varepsilon_1 - \varepsilon}{3\varepsilon} \vec{E}$$

Radiation from an oscillating dipole

Hertzian dipole
(Rayleigh)



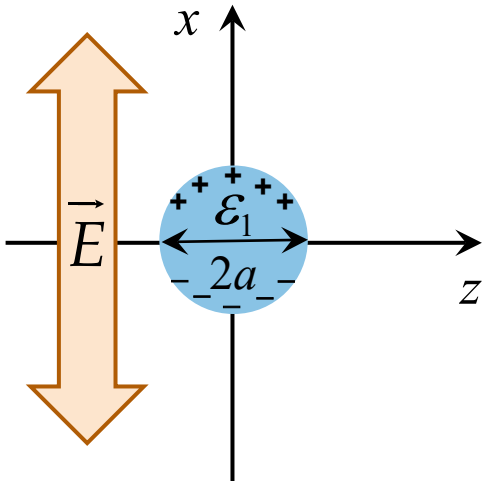
Dipole moment: $\vec{p} = q\,d\,\hat{x}$

Radiated E-far field:

$$\vec{E}_{far-field} = \frac{j\,k\,i\,d}{4\pi\varepsilon c\,r}\sin\theta\,\mathrm{e}^{-jkr}\,\hat{\theta}$$

Current: $i = j\omega q \;\Rightarrow\; i\,d = j\omega\left|\vec{p}\right|$

Particle dipole
(Einstein-Smoluchowski)

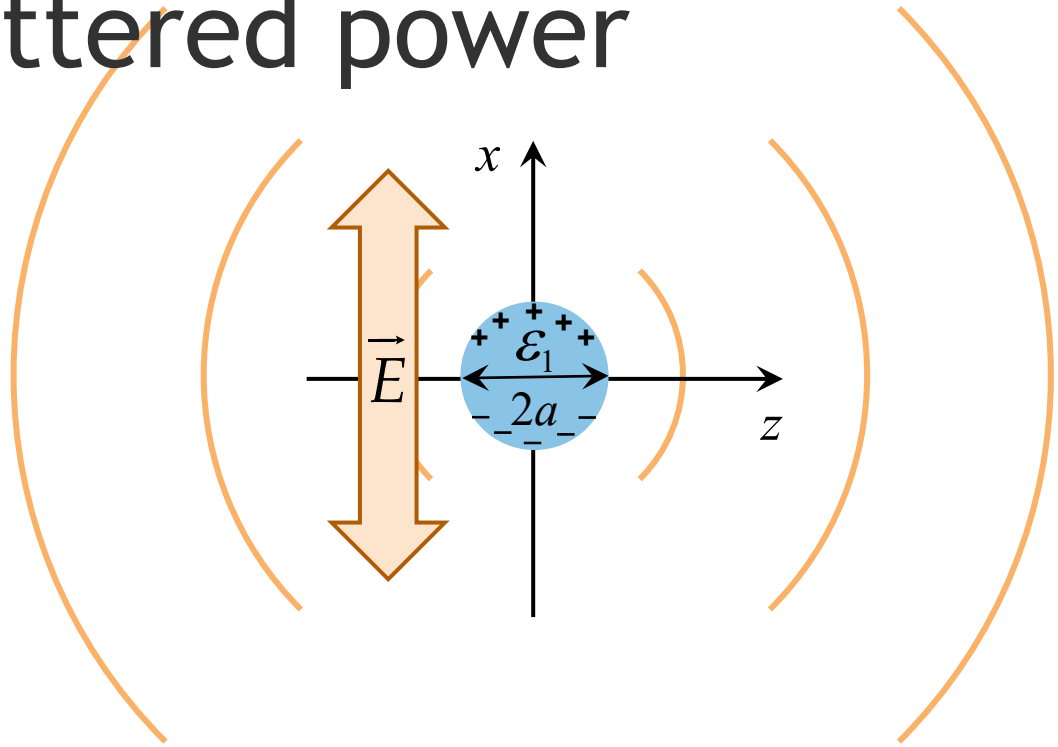


Dipole moment: $\vec{p} = 4\pi\varepsilon\,a^3\,\dfrac{\varepsilon_1 - \varepsilon}{3\varepsilon}\,\vec{E}$

Radiated E-far field:

$$\vec{E}_{far-field} = -\frac{k^2 a^3}{r}\,\frac{\varepsilon_1 - \varepsilon}{3\varepsilon}\left|\vec{E}\right|\sin\theta\,\mathrm{e}^{-jkr}\,\hat{\theta}$$

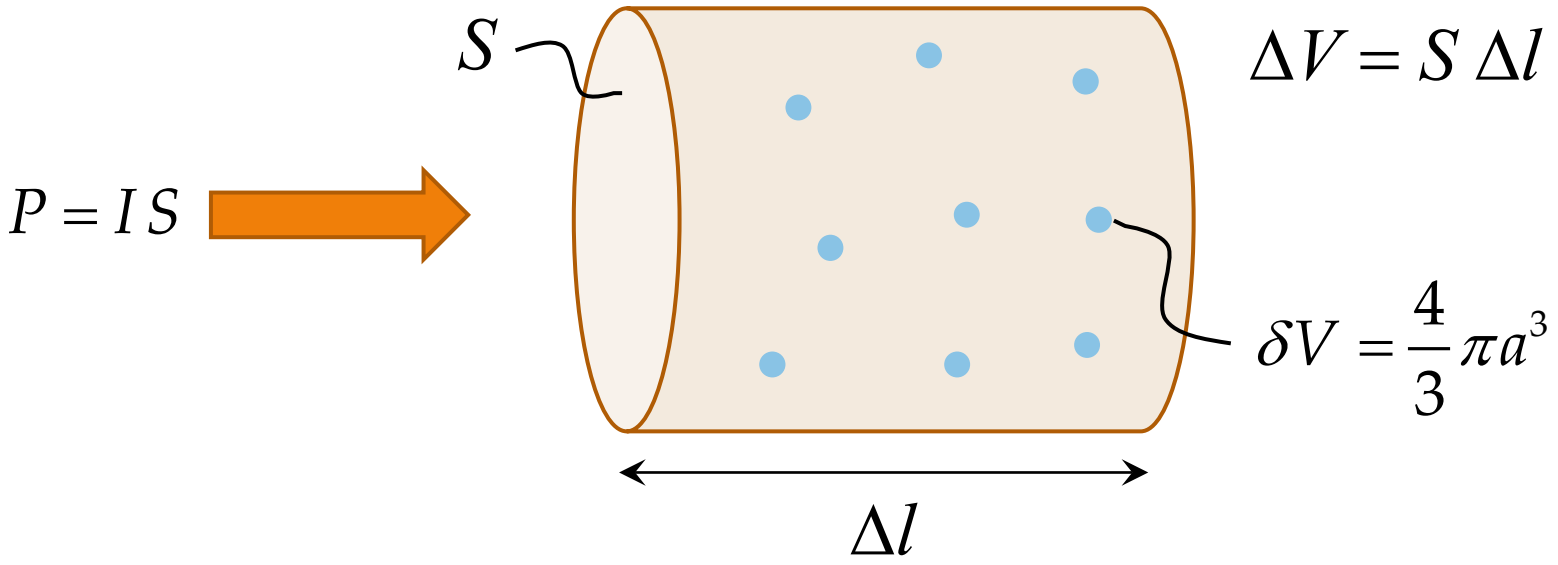
Total scattered power



$$P_s = \int_0^{2\pi}\int_0^{\pi} \varepsilon c \frac{\left|\vec{E}_{far-field}\right|^2}{2} r^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{4\pi\varepsilon c}{3} k^4 a^6 \left(\frac{\varepsilon_1 - \varepsilon}{3\varepsilon}\right)^2 \left|\vec{E}\right|^2 = \frac{8\pi}{3} k^4 a^6 \left(\frac{\varepsilon_1 - \varepsilon}{3\varepsilon}\right)^2 I \quad \text{with} \quad I = \frac{\varepsilon c}{2}\left|\vec{E}\right|^2$$

Scattering loss coefficient α_s



Let assume the scattering particles densely packed (no gap between particles)

➔ Particles density (number per unit volume) $N = 1 / \delta V$

Power loss in the volume ΔV: $\Delta P = -\alpha_s\, P\, \Delta l$ ➔ $-N\, P_s\, \Delta V = -\alpha_s\, P\, \Delta l$

$-N\, P_s\, S\, \Delta l = -\alpha_s\, P\, \Delta l$

➔ $$\alpha_s = N\,S\frac{P_s}{P} = \frac{1}{\delta V}\frac{P_s}{I} = \frac{8}{3}\frac{\pi^3}{\lambda_o^4}\left\langle (\Delta\varepsilon)^2 \right\rangle \delta V$$

with $\Delta\varepsilon = \varepsilon_1 - \varepsilon$

Scattering loss coefficient α_s

$$\alpha_s = \frac{8}{3}\frac{\pi^3}{\lambda_o^4}\left\langle (\Delta\varepsilon)^2 \right\rangle \delta V$$

$\Delta\varepsilon$ is function of the material density ρ and the temperature T:

$$\Delta\varepsilon = \left(\frac{\partial\varepsilon}{\partial\rho}\right)_T \Delta\rho + \left(\frac{\partial\varepsilon}{\partial T}\right)_\rho \Delta T$$

Normally for solid state materials: $\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T >> \left(\frac{\partial\varepsilon}{\partial T}\right)_\rho$

$$\rightarrow \left\langle (\Delta\varepsilon)^2 \right\rangle \cong \left(\frac{\partial\varepsilon}{\partial\rho}\right)_T^2 \left\langle (\Delta\rho)^2 \right\rangle$$

From the theory of thermodynamic fluctuations: $\left\langle (\Delta\rho)^2 \right\rangle = \frac{\rho^2}{\delta V} k_B T_f \beta_c$

T_f is a fictitious temperature representing the temperature when all density fluctuations get frozen (~solidification).

β_c is the isothermal compressibility: $\beta_c = -\frac{1}{\delta V}\left(\frac{\partial \delta V}{\partial P}\right)_T = 6.9\ 10^{-11}\ \frac{\mathrm{m}^2}{\mathrm{J}}$ in SiO^2

Scattering loss coefficient α_s

$$\alpha_s = \frac{8}{3}\frac{\pi^3}{\lambda_o^4}\left(\rho\frac{\partial\varepsilon}{\partial\rho}\right)_T^2 k_B T_f \beta_c$$

Since silica is a isotropic solid state material, the following term can be made explicit using the theory of photoelasticity:

$$\left(\rho\frac{\partial\varepsilon}{\partial\rho}\right)_T = n^4 p_{12}$$

n: the refractive index; $p_{12} = 0.286$ photoelastic coefficient

It is also possible to use the Clausius-Mossoti relation.

➔ **Rayleigh scattering loss coefficient:** $\alpha_s = \frac{8}{3}\frac{\pi^3}{\lambda_o^4} n^8 p_{12}^2 k_B T_f \beta_c$

It is better to choose for a material with low refractive index and compressibility.

2 main contributions for Rayleigh scattering in optical fibres

Silica glass is characterized by **nanometric fluctuations** of $\varepsilon \Rightarrow$ Rayleigh scattering.

Local density fluctuation frozen in the glass during cooling

$$\alpha_{Rd} = \frac{8}{3}\frac{\pi^3}{\lambda_o^4} n^8 p_{12}^2 k_B T_f \beta_c$$

The critical parameter is the fictitious temperature

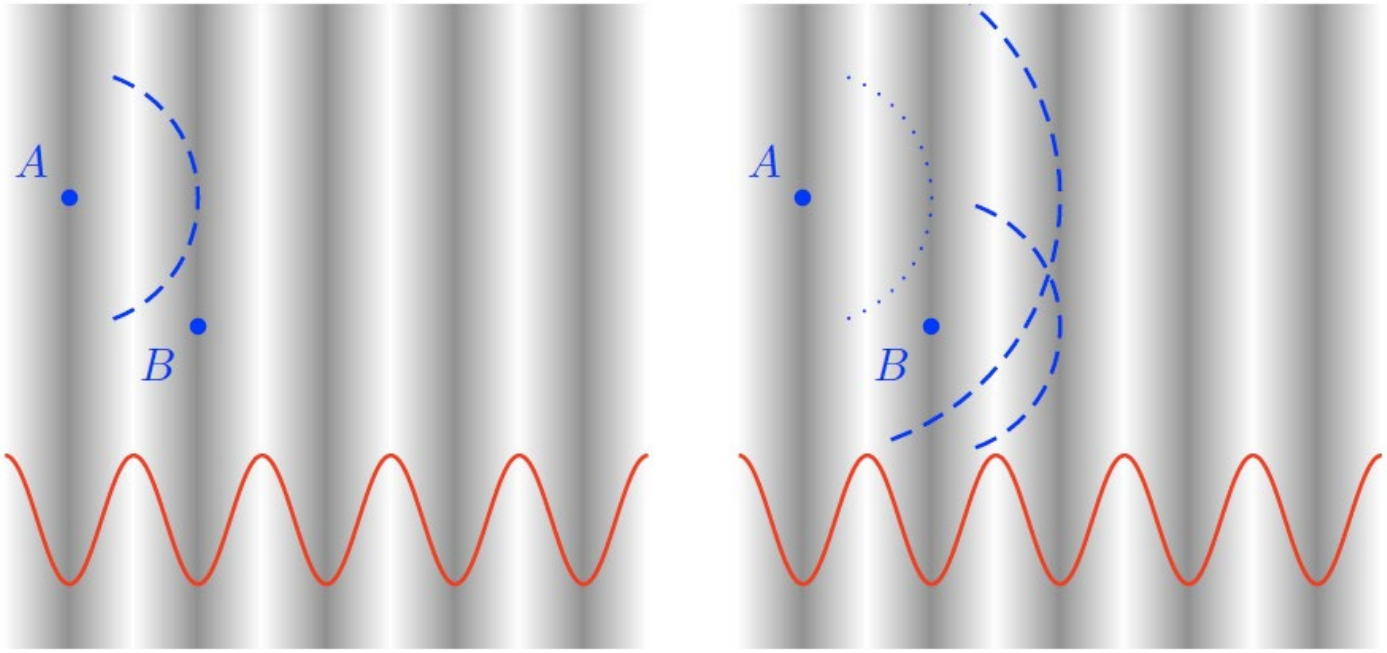
Local fluctuations of composition (doping)

$$\alpha_{Rc} = \frac{8}{3}\frac{\pi^3}{\lambda_o^4}\left(\frac{\partial n^2}{\partial N}\right)^2 \left\langle (\delta N)^2 \right\rangle$$

N is the number of doping molecules per unit of volume

Credit to Prof. Marc Wuilpart, University of Mons, Belgium

In the forward direction, all the scattered wavelets sum up constructively



- The incident light is assumed to be **locally a plane wave**. A and B are two random scattering centres.
- **The wave scattered by B is necessarily in phase with the wave priorily scattered by A**. Extrapolated to a large number of centres, all scattered waves show a **constant phase relationship** with the incident wave. The phase lag between forward scattered waves and incident wave is an interpretation of the wave propagation slowing (**refractive index**).

Credit to Prof. Marc Wuilpart, University of Mons, Belgium

There are two main components to scalar scattering

$$\Delta\rho = \left(\frac{\partial\rho}{\partial p}\right)\Delta p + \left(\frac{\partial\rho}{\partial s}\right)\Delta s$$

- Adiabatic density fluctuations
- **Pressure fluctuations mainly results from acoustical waves propagation ➔ described by a wave equation**
- The scattered wave is frequency-shifted
- Inelastic scattering
- **Brillouin scattering**

- Isobaric density fluctuations
- **Entropy fluctuations are described by a diffusion equation**
- The scattered wave is not frequency-shifted
- Elastic scattering
- **Rayleigh scattering**

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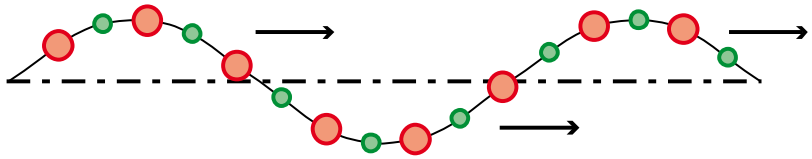
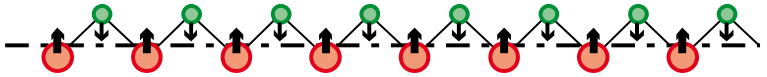
Physics behind inelastic scatterings

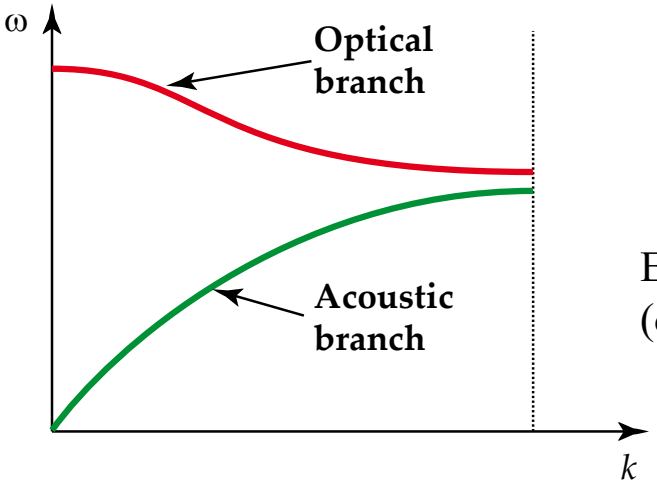
In a solid state constituted of *polyatomic* molecules, the cohesive force between molecules allows a collective vibration into two distinct vibrational modes:



- *Oscillatory* movement of the entire molecular chain.
- *Classical* wave, *slow* vibration transporting *high momentum*.
- *Acoustic*-like vibration.



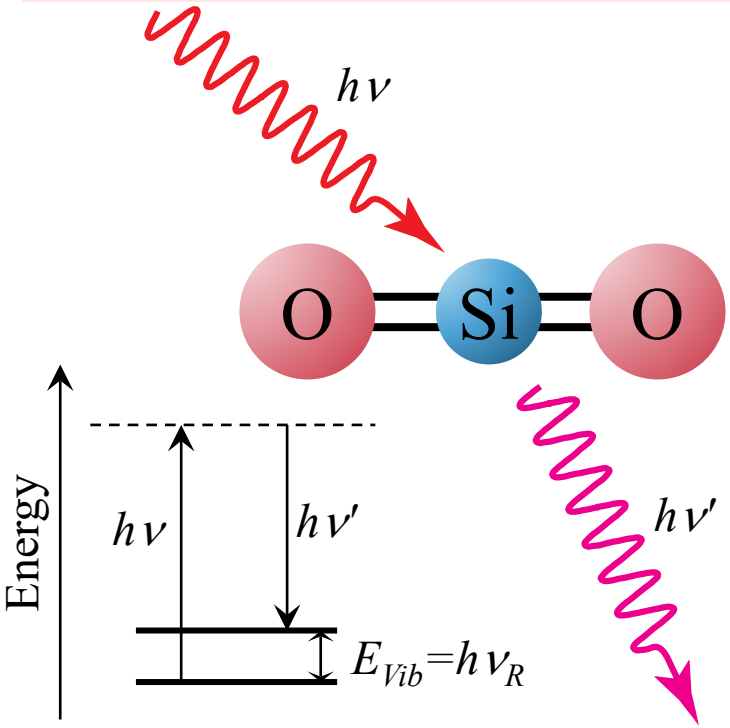
- *Vibrational* oscillation inside the molecular chain.
- *Quantum* excited state, *fast* vibration with *small momentum*.
- *Optical*-like vibration.

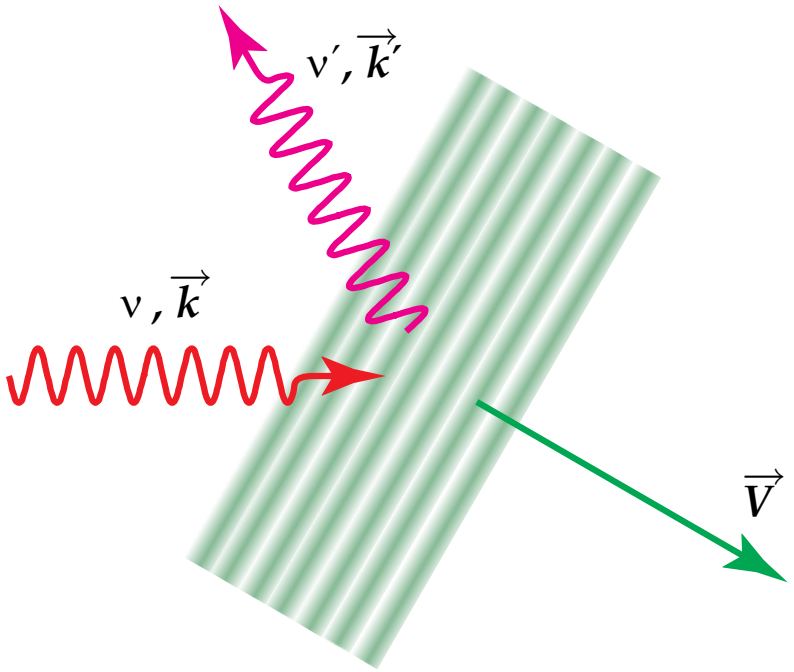


Energy-momentum diagram (dispersion curve)

Optical effect of inelastic scatterings

Optical branch:	**Raman scattering** **High energy** phonons with **low momentum** **Large** spectral shift (~12 THz or 96 nm at λ_o=1550 nm in SiO_2) and **non-strict** phase matching.	
Acoustic branch:		**Brillouin scattering** **Low energy** phonons with **high momentum** **Small** spectral shift (~11 GHz or 0.07 nm at λ_o=1550 nm in SiO_2) and **strict** phase matching.





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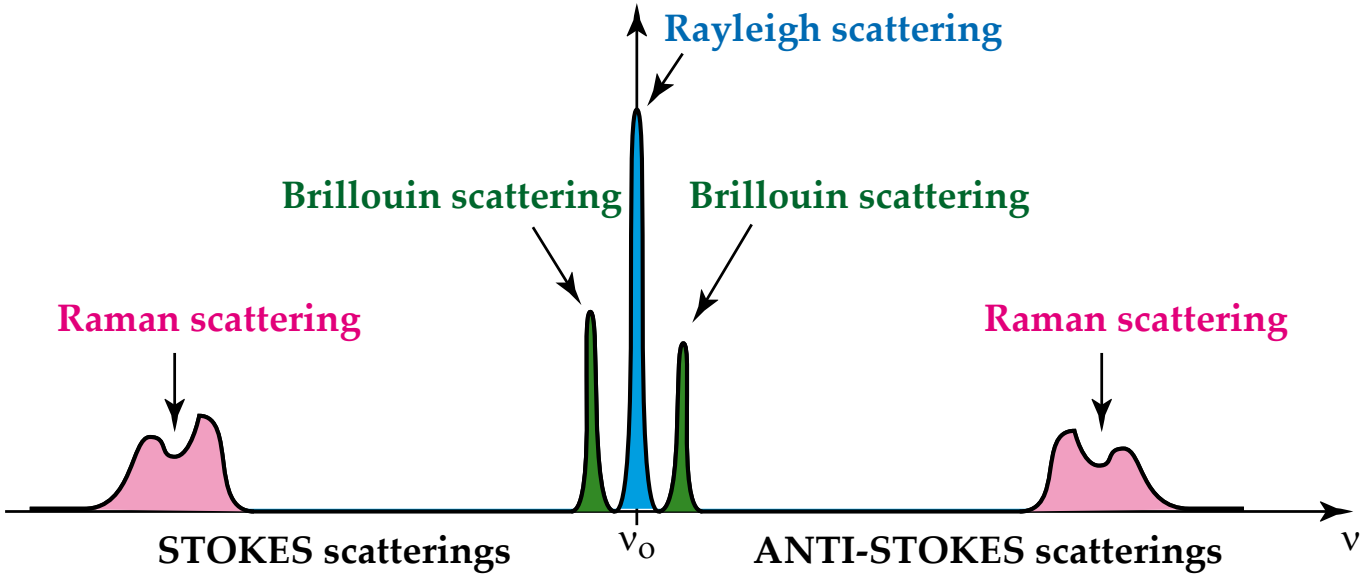


Spontaneous inelastic scatterings

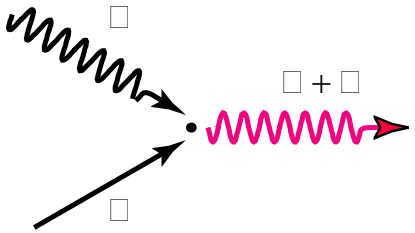
Spontaneous inelastic scatterings are **purely thermally activated** and are thus **linear** processes.

Spontaneous inelastic scatterings are generated by **thermal phonons**
➔ The average number of phonons is governed by **Bose-Einstein statistics**

$$\bar{n} = \frac{1}{e^{\frac{h\Omega}{kT}} - 1}$$

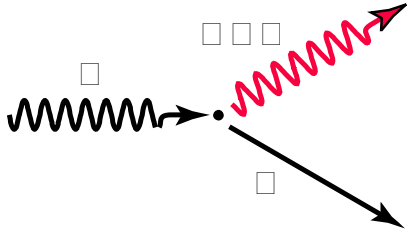
- **Anti-Stokes** scattering annihilates a phonon
  ➔ Scattering coefficient is proportional to $\bar{n}$

$$C_{AS} \sim \bar{n} = \frac{1}{e^{\frac{h\Omega}{kT}} - 1}$$



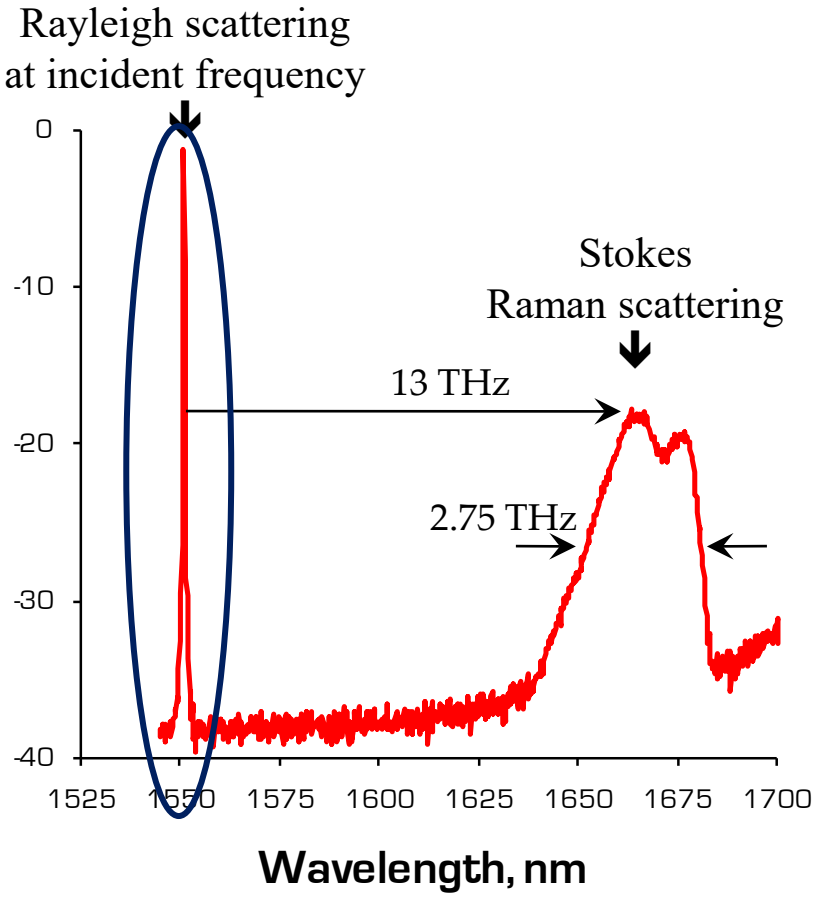
- **Stokes** scattering creates a phonon
  ➔ Scattering coefficient proportional to $\bar{n}+1$

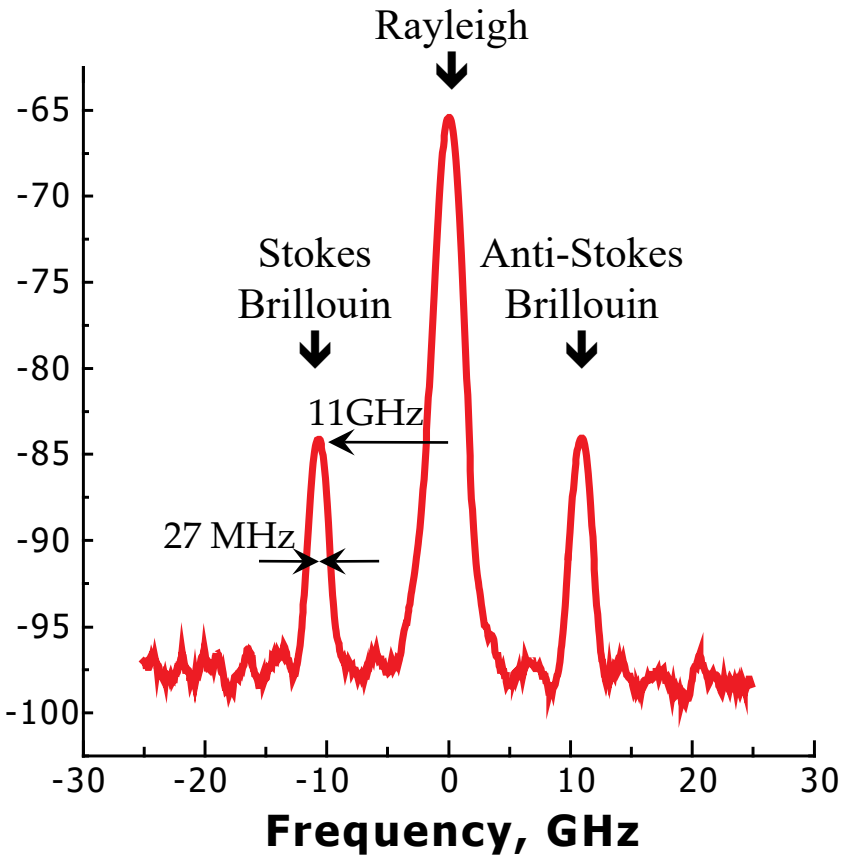
$$C_S \sim \bar{n} + 1 = \frac{e^{\frac{h\Omega}{kT}}}{e^{\frac{h\Omega}{kT}} - 1}$$



	Stokes shift Ω	Average phonon number	
Raman	13.2 THz	0.14	Anti-Stokes < Stokes
Brillouin	11 GHz	570	Anti-Stokes ~ Stokes

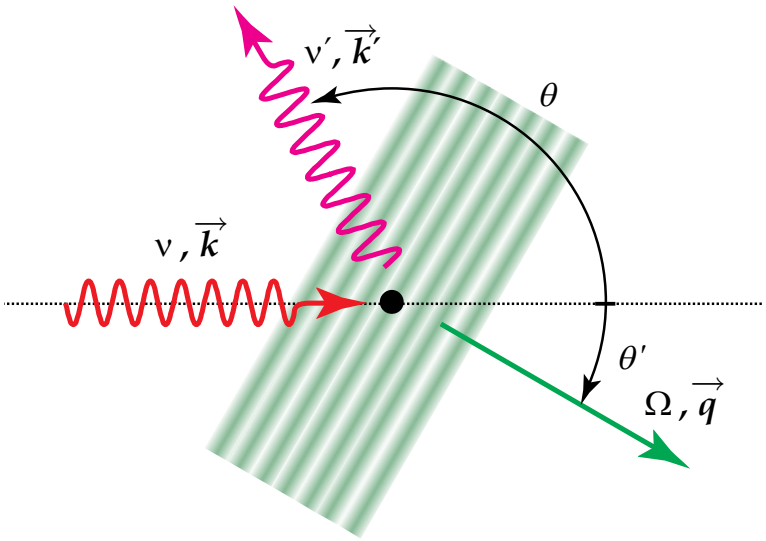
Spectral characteristics of inelastic scatterings





The *linewidth* of Brillouin scattering is ruled by the *acoustic loss* (lifetime ~6ns)

Brillouin: Energy-momentum conservation



Energy conservation:

$$h\nu = h\nu' + h\Omega$$

Momentum conservation:

$$\hbar\vec{k} = \hbar\vec{k}' + \hbar\vec{q}$$

Splitting this vectorial equation into components:

$$\begin{cases} k'\sin\theta = q\sin\theta' \\ k = k'\cos\theta + q\cos\theta' \end{cases}$$

Squaring and combining these equations using the relationships $\nu = kc/2\pi$, $\nu' = k'c/2\pi$, $\Omega = qV_a/2\pi$ and the energy conservation $\nu' = \nu - \Omega$:

$$4V_a^2\nu(\nu-\Omega)\sin^2\frac{\theta}{2} = (c^2 - V_a^2)\Omega^2$$

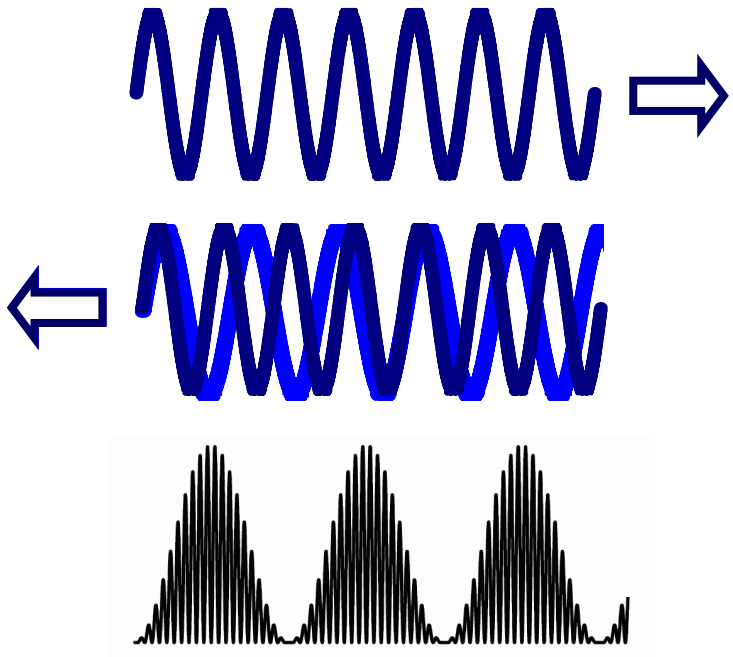
Solving for Ω yields: $\Omega = 2\frac{V_a}{c}\nu\sin\frac{\theta}{2}\sqrt{1-\left(\frac{V_a}{c}\right)^2\cos^2\frac{\theta}{2}} - 2\left(\frac{V_a}{c}\right)^2\nu \overset{V_a \ll c}{\simeq} 2\frac{V_a}{c}\nu\sin\frac{\theta}{2} = 2n\frac{V_a}{\lambda_o}\sin\frac{\theta}{2}$

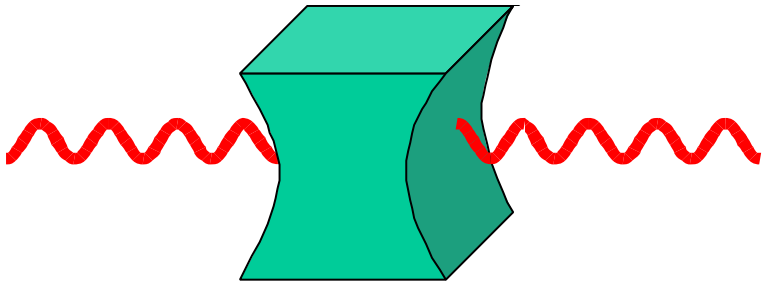
2 possibilities in single mode fibres: $\theta=0$ ➔ $\Omega=0$ No effect, *no forward scattering*!

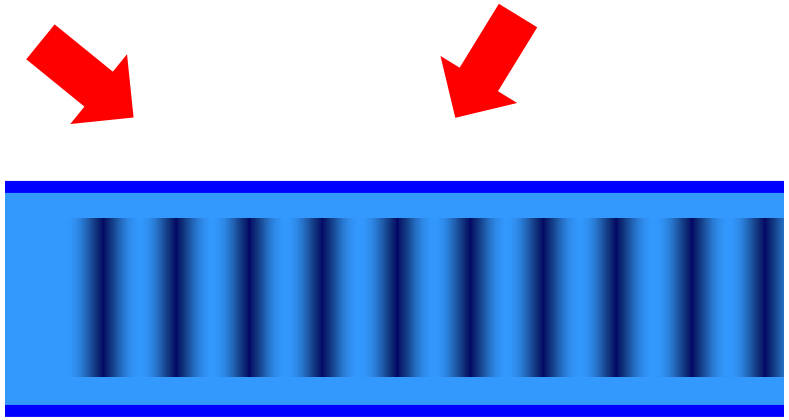
$\theta=\pi$ ➔ $\Omega=2nV_a/\lambda_o$ **Backward scattering**, max. frequency shift

Mechanisms behind stimulated scatterings

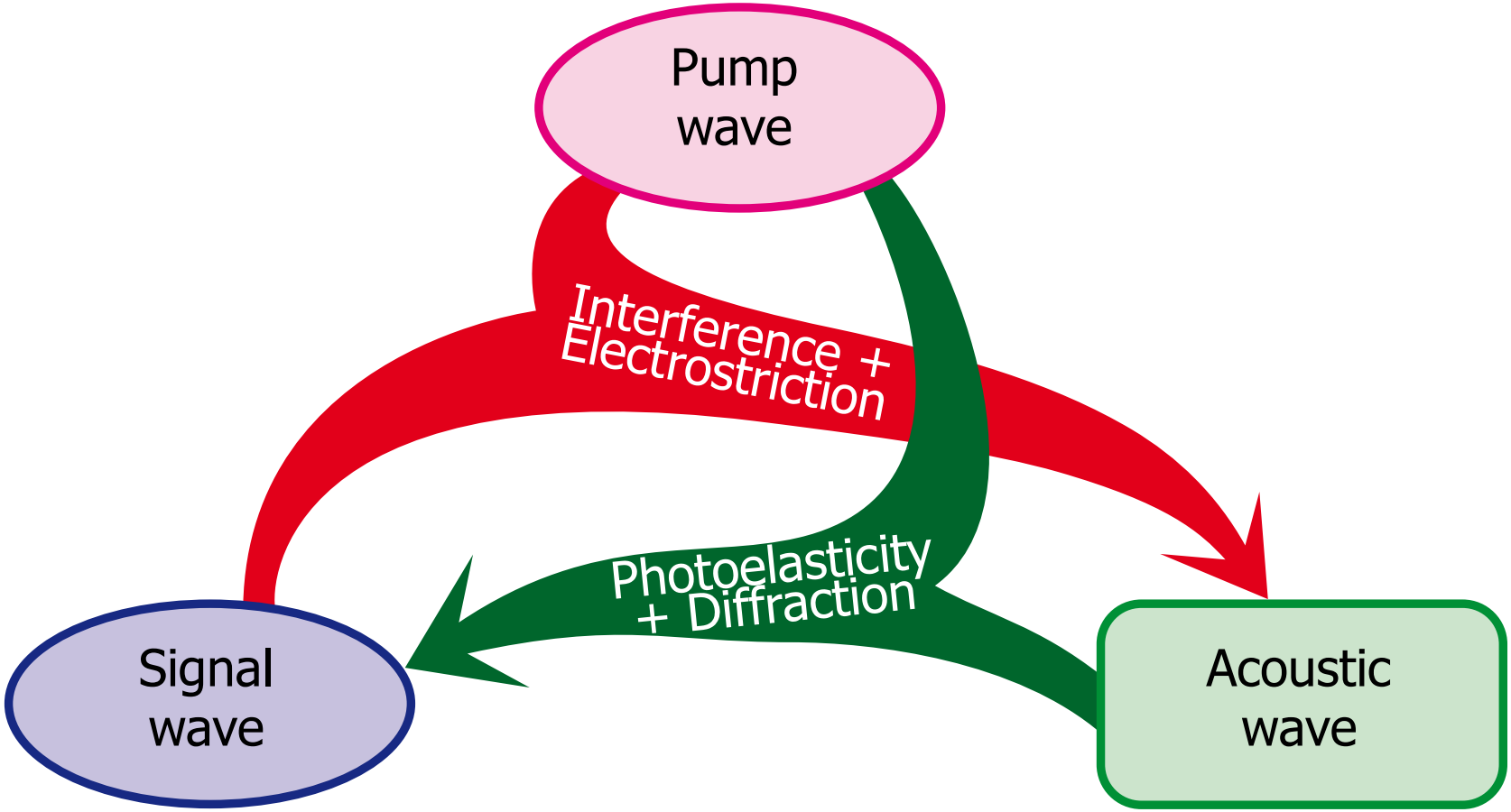
Interferences + **Electrostriction**







Principle of Brillouin stimulated scattering



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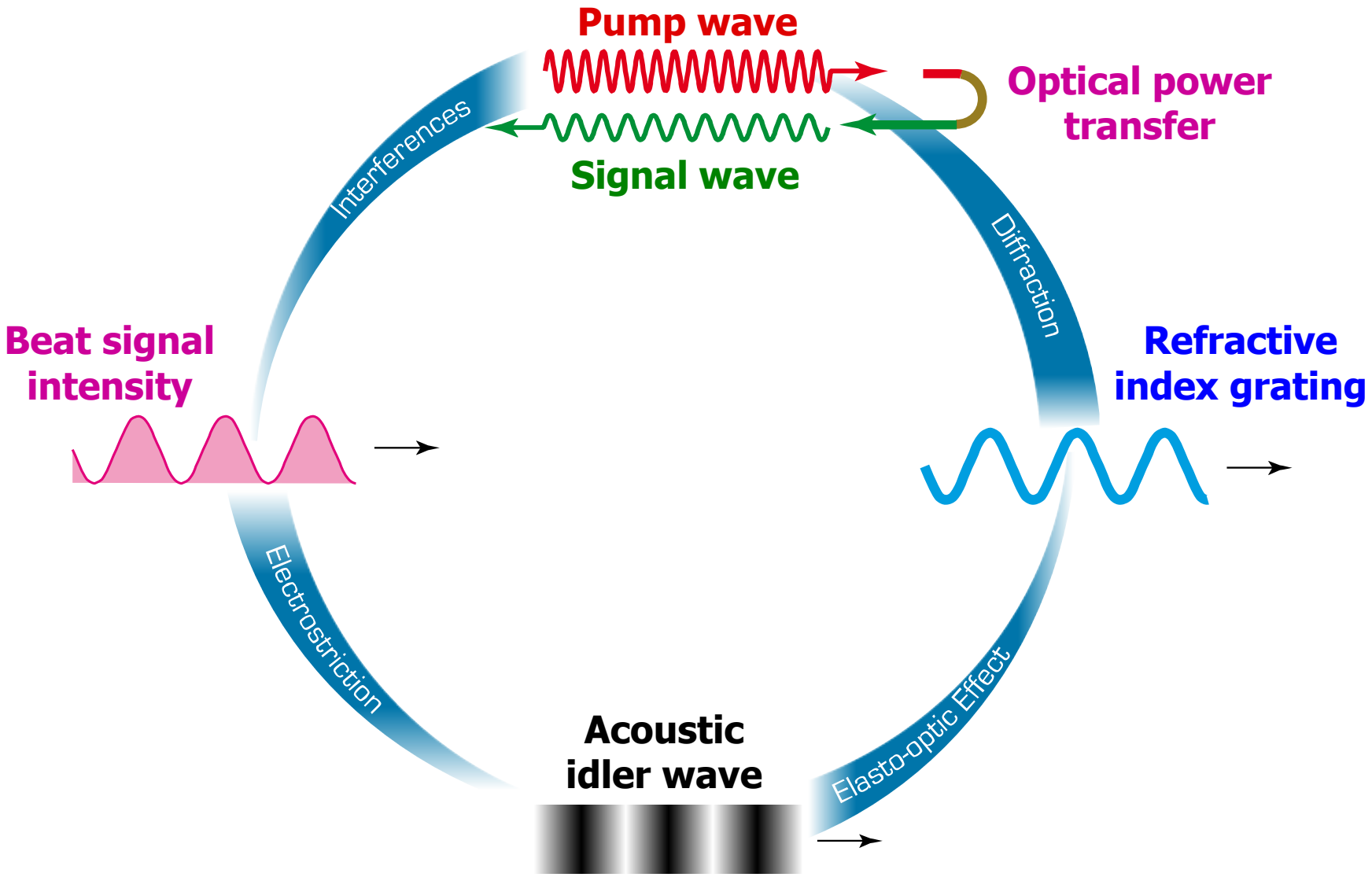

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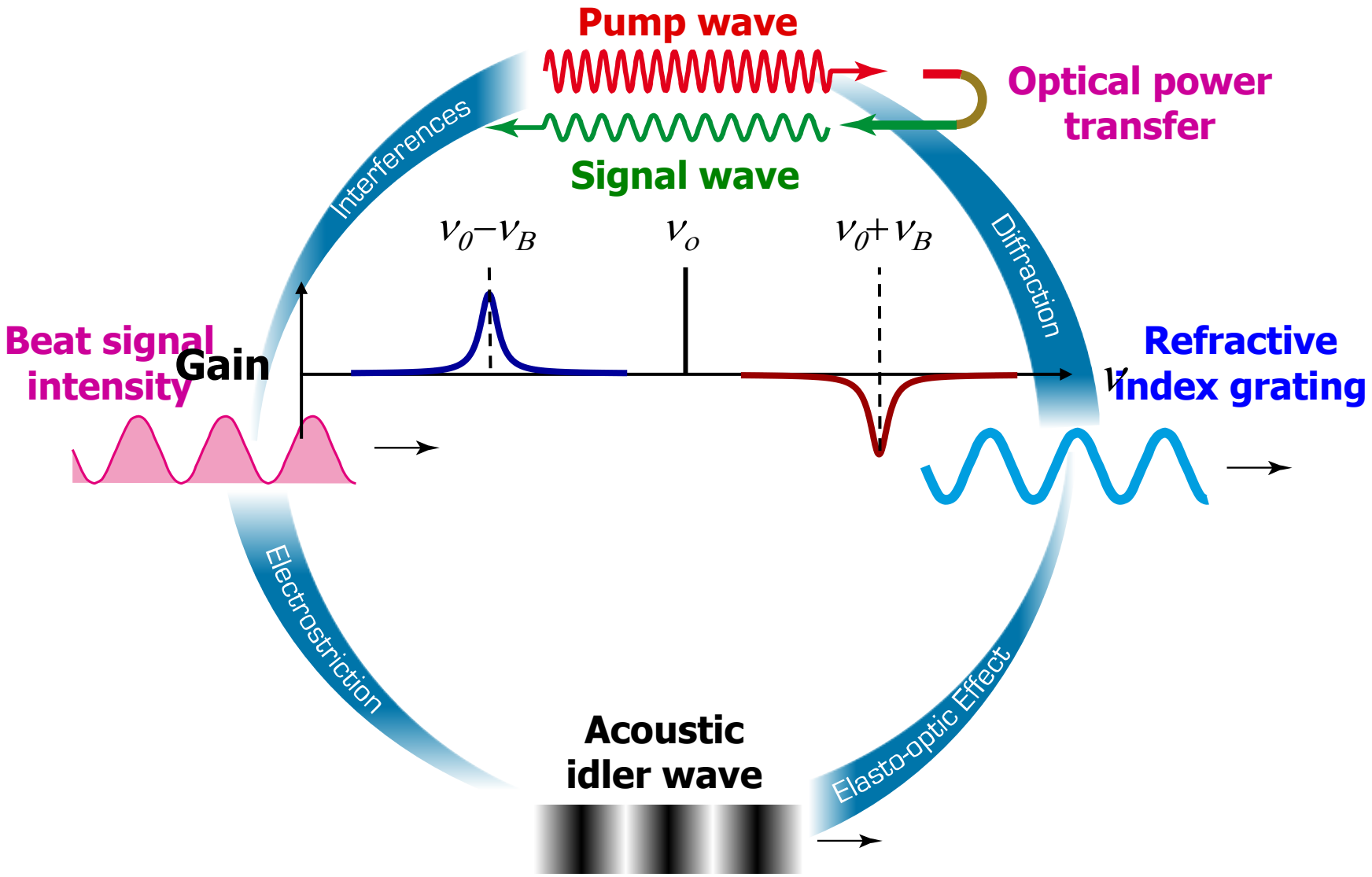
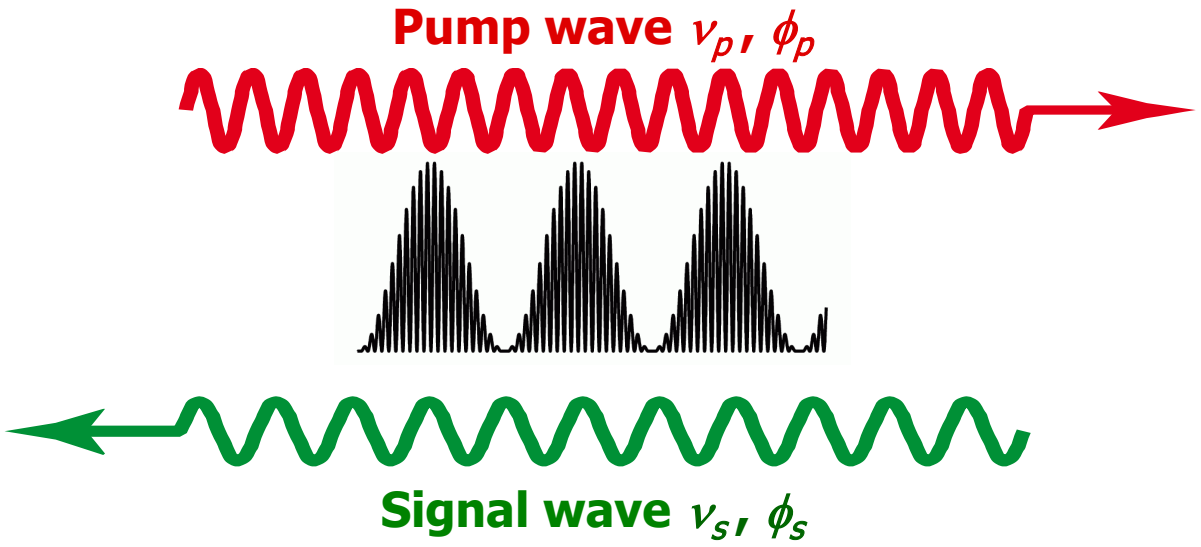


Grating picture of Brillouin scattering

In *steady state* conditions the *grating* formed by the acoustic wave through the *interference* of the pump and signal waves automatically creates a *matched coupling*.



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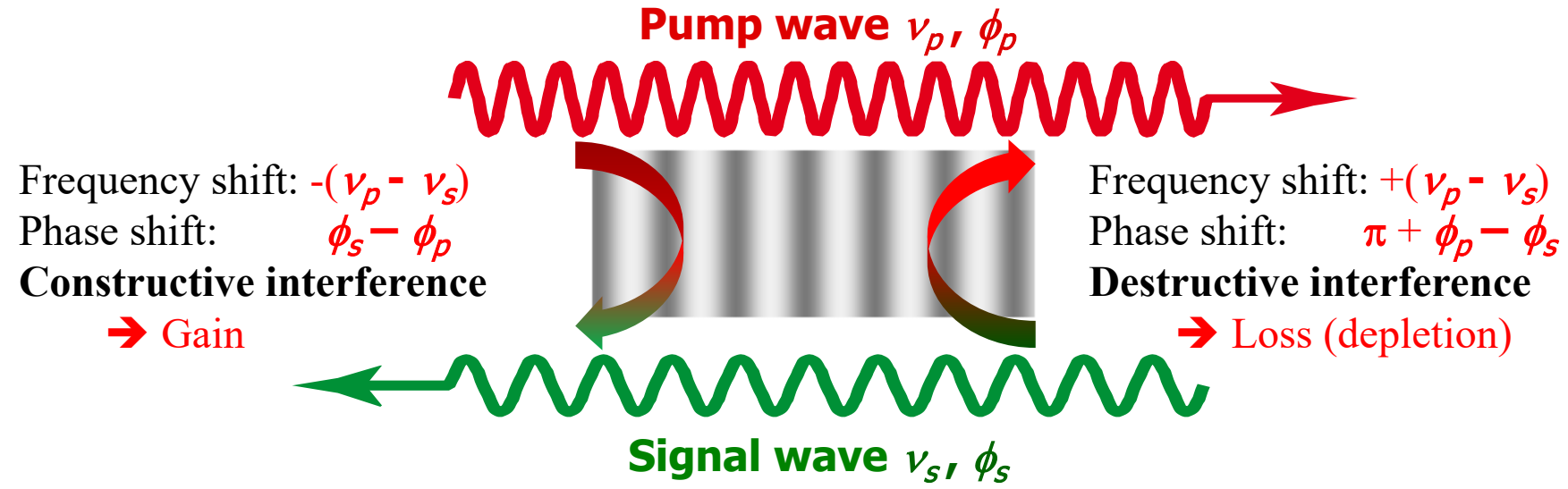


All wave properties are properly *transformed*, except:

- *Polarization* of the incident wave is preserved
- The coupling can be *temporarily unmatched* due to the inertial response of the acoustic wave (τ~12 ns).

Stimulated Raman scattering

For a *forward* propagating pump and a *backward* signal the interaction is governed by the following set of coupled equations:

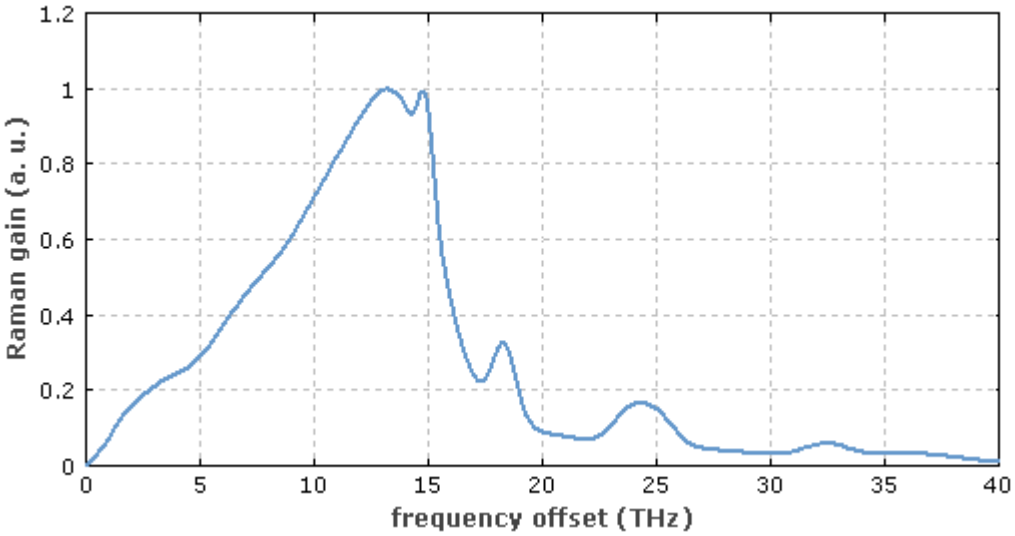
$$\begin{cases} \dfrac{dI_P}{dz} = -\dfrac{\nu_P}{\nu_S} g_R I_P I_S - \alpha_P I_P \\ \dfrac{dI_S}{dz} = -g_R I_P I_S + \alpha_S I_S \end{cases}$$

In absence of *pump depletion*:

$$I_S(L) = I_S(0)\, e^{g_R I_P(0) L_{eff} - \alpha_S L}$$

with $L_{eff} = (1 - e^{-\alpha_P L}) / \alpha_P$

Nonlinear effective length



$$g_R \simeq 10^{-13} \frac{\text{m}}{\text{W}}$$

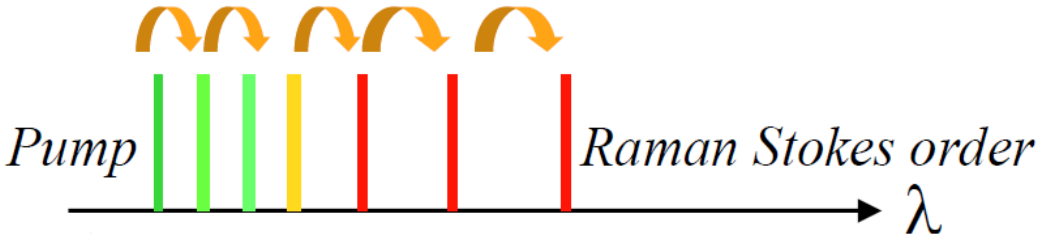
1 W of pump power through 1 km of fibre

⬇

Net gain = 1.6

Cascaded Raman generation

- Each generated Stokes wave can act as a pump to generate an additional order



Stimulated Brillouin scattering

For a *forward* propagating pump and a *backward* signal the interaction is governed by the following set of coupled equations for the amplitudes:

$$\begin{cases} \dfrac{\partial A_p}{\partial z}+\dfrac{1}{V_g}\dfrac{\partial A_p}{\partial t}=i\dfrac{1}{2}g_2 A_s Q-\dfrac{\alpha}{2}A_p \\ \dfrac{\partial A_s}{\partial z}-\dfrac{1}{V_g}\dfrac{\partial A_s}{\partial t}=-i\dfrac{1}{2}g_2 A_p Q^*+\dfrac{\alpha}{2}A_s \\ \dfrac{\partial Q}{\partial t}+\Gamma_A Q=ig_1 A_p A_s^* \end{cases} \qquad \text{with} \quad \Gamma_A=i\frac{2\pi(\nu_B^2-\Omega^2)-i\Omega\Gamma_B}{2\Omega}$$

Frequency detuning factor including a loss $\Gamma_B=1/\tau_A$

τ_A = Acoustic lifetime

Steady-state situation $(T>>\tau_A)$:

$$Q=i\frac{g_1}{\Gamma_A}A_p A_s^* \Rightarrow \begin{cases} \dfrac{\partial A_p}{\partial z}=-\dfrac{1}{2}\dfrac{g_1 g_2}{\Gamma_A}\left|A_s\right|^2 A_p-\dfrac{\alpha}{2}A_p \\ \dfrac{\partial A_s}{\partial z}=-\dfrac{1}{2}\dfrac{g_1 g_2}{\Gamma_A}\left|A_p\right|^2 A_s+\dfrac{\alpha}{2}A_s \end{cases} \Rightarrow \begin{cases} \dfrac{\partial I_p}{\partial z}=-g_1 g_2\,\mathrm{Re}\{\dfrac{1}{\Gamma_A}\}I_s I_p-\alpha I_p \\ \dfrac{\partial I_s}{\partial z}=-g_1 g_2\,\mathrm{Re}\{\dfrac{1}{\Gamma_A}\}I_p I_s+\alpha I_s \end{cases}$$

Stimulated Brillouin scattering

$$\begin{cases} \dfrac{\partial I_p}{\partial z} = -g_1 g_2 \,\mathrm{Re}\{\dfrac{1}{\Gamma_A}\} I_s I_p - \alpha I_p \\ \dfrac{\partial I_s}{\partial z} = -g_1 g_2 \,\mathrm{Re}\{\dfrac{1}{\Gamma_A}\} I_p I_s + \alpha I_s \end{cases}$$

$$\Rightarrow \quad g_1 g_2 \,\mathrm{Re}\{\frac{1}{\Gamma_A}\} = \frac{4 g_1 g_2}{\Gamma_B} \frac{(\Gamma_B/2\pi)^2}{(\nu_B - \Omega)^2 + (\Gamma_B/2\pi)^2} = g_B \frac{(\Gamma_B/2\pi)^2}{(\nu_B - \Omega)^2 + (\Gamma_B/2\pi)^2} = g(\Omega)$$

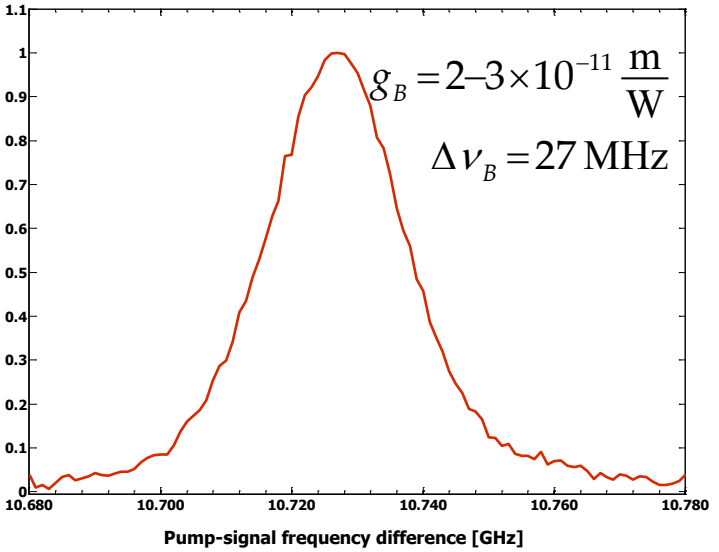
Lorentzian gain spectrum with $g_B = 4 g_1 g_2 / \Gamma_B$ and $\Delta\nu_B = \Gamma_B/\pi = 1/\pi\tau_A$

In absence of *pump depletion*:

$$I_S(L) = I_S(0)\, \mathrm{e}^{g_B I_P(0) L_{eff} - \alpha L}$$

with $L_{eff} = (1 - \mathrm{e}^{-\alpha L}) / \alpha$

Nonlinear effective length



Brillouin amplifier is a linear system

➔ 2 *appended* fibers is equivalent to 1 fiber with the *summed* gain spectra:

$g_{B1}(\nu)\ ,\ L_1$ $\qquad$ $g_{B2}(\nu)\ ,\ L_2$

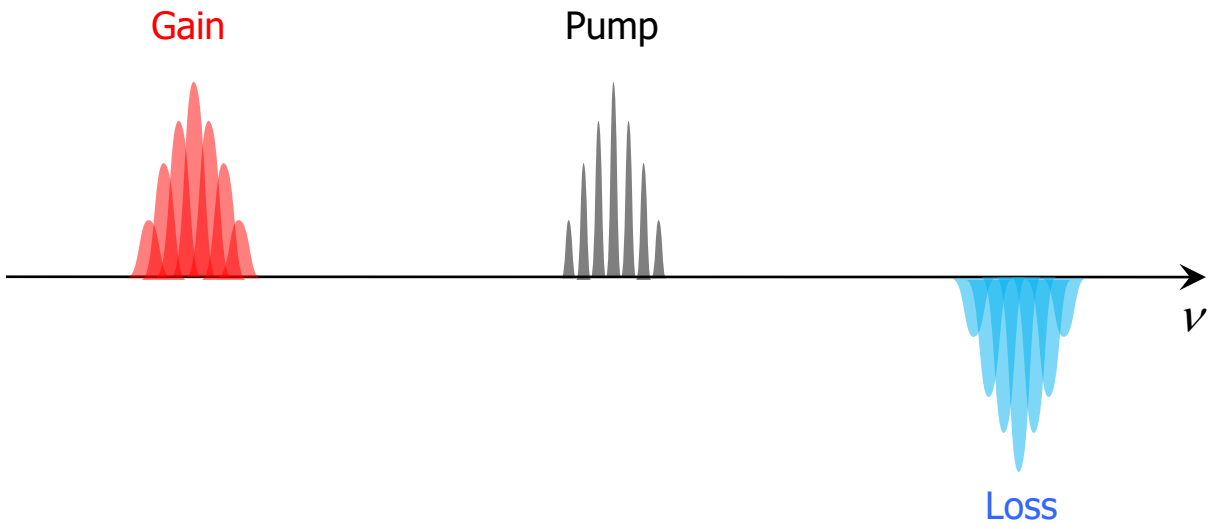
$$I_{out}(\nu) = e^{g_{B2}(\nu)\, I_{Pump} L_2}\, e^{g_{B1}(\nu)\, I_{Pump} L_1}\, I_{in}(\nu) = e^{[g_{B1}(\nu)\frac{L_1}{L} + g_{B2}(\nu)\frac{L_2}{L}]\, I_{Pump} L}\, I_{in}(\nu) \quad , \quad L = L_1 + L_2$$

➔ 2 *pumps* in 1 fibre is equivalent to 1 pump with the *summed* gain spectra:

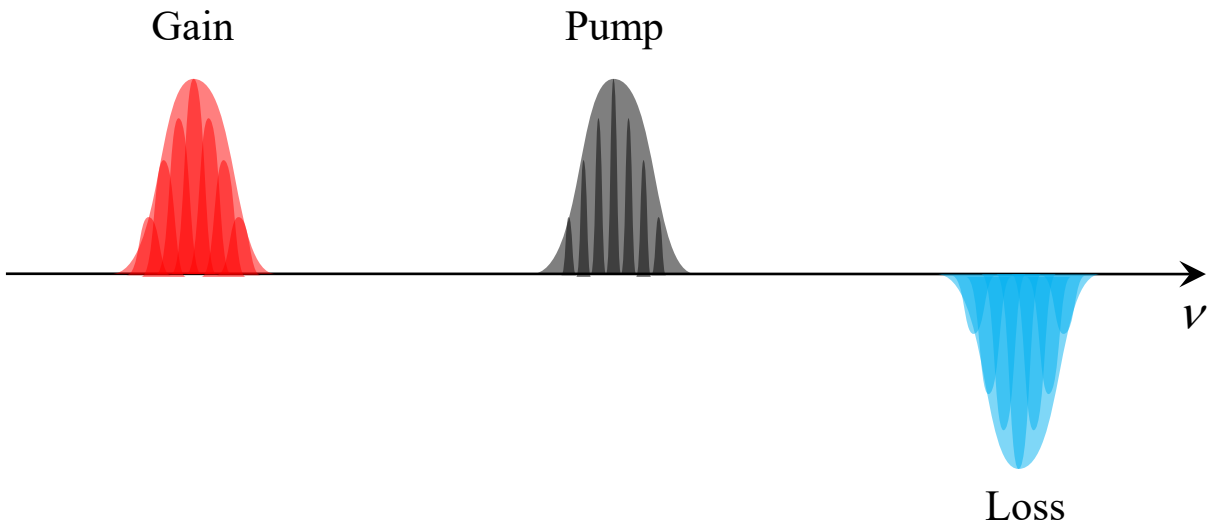
$$I_{out}(\nu) = e^{g_{B1}(\nu)\, I_{P1} L + g_{B2}(\nu)\, I_{P2} L}\, I_{in}(\nu) = e^{[g_{B1}(\nu)\frac{I_{P1}}{I_{Pump}} + g_{B2}(\nu)\frac{I_{P2}}{I_{Pump}}]\, I_{Pump} L}\, I_{in}(\nu), \quad I_{Pump} = I_{P1} + I_{P2}$$

This description can be extended to N fibers and N pumps

Broadband Brillouin gain by pump modulation



Broadband Brillouin gain by pump modulation

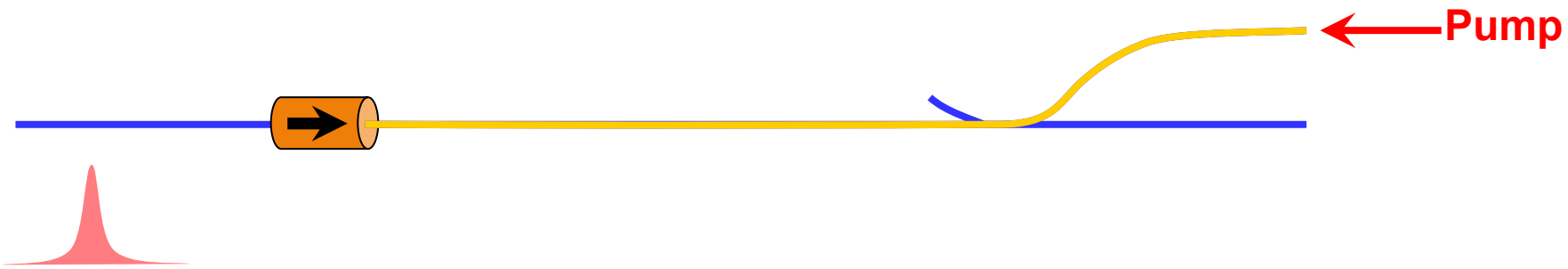


The *effective* gain/loss spectrum can be shaped by properly *modulating* the pump

$$g_B^{eff}(\nu) = g_B(\nu) \otimes \frac{I_p(\nu)}{I_{pump}}$$

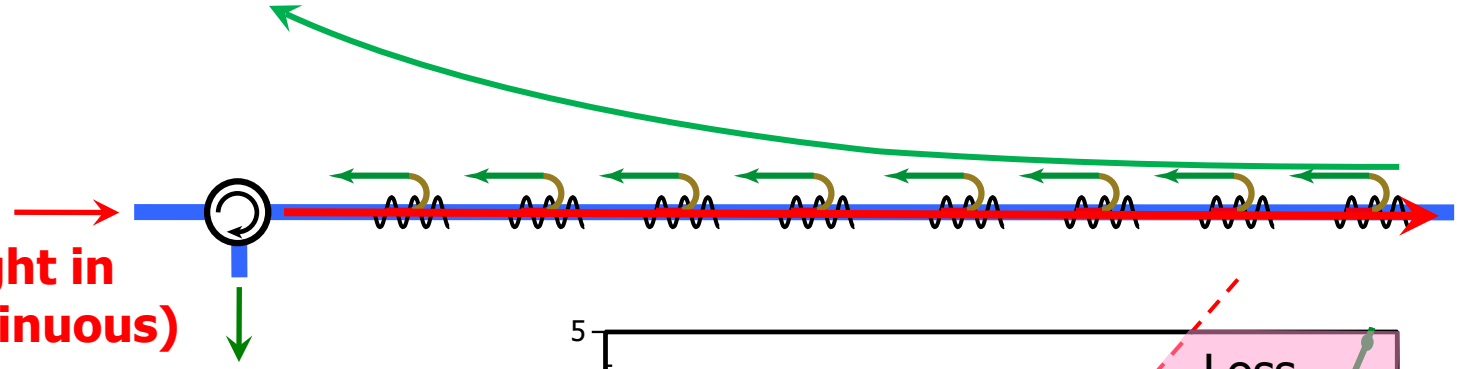
Convolution between the natural Brillouin gain and the pump spectra

Amplifiers based on stimulated scatterings



- Stimulated scatterings may realise the closest approximation of *fully distributed amplification*, considered as an ideal case.
- Raman-based amplifiers are very attractive: large *bandwidth*, *wavelength flexibility* and *low noise*. But: low gain ➔ *high pump power*
- Brillouin-based amplifiers also show a good *wavelength flexibility* and give a much *higher gain*, but very *limited bandwidth* and *poor noise* figure.
- Noise is determined by the amount of *spontaneous scattering*, which is scaled by the *average number* of *thermally-created phonons* (0.14 per mode for Raman scattering, 570 per mode for Brillouin scattering).

Brillouin limits the power handling capacity

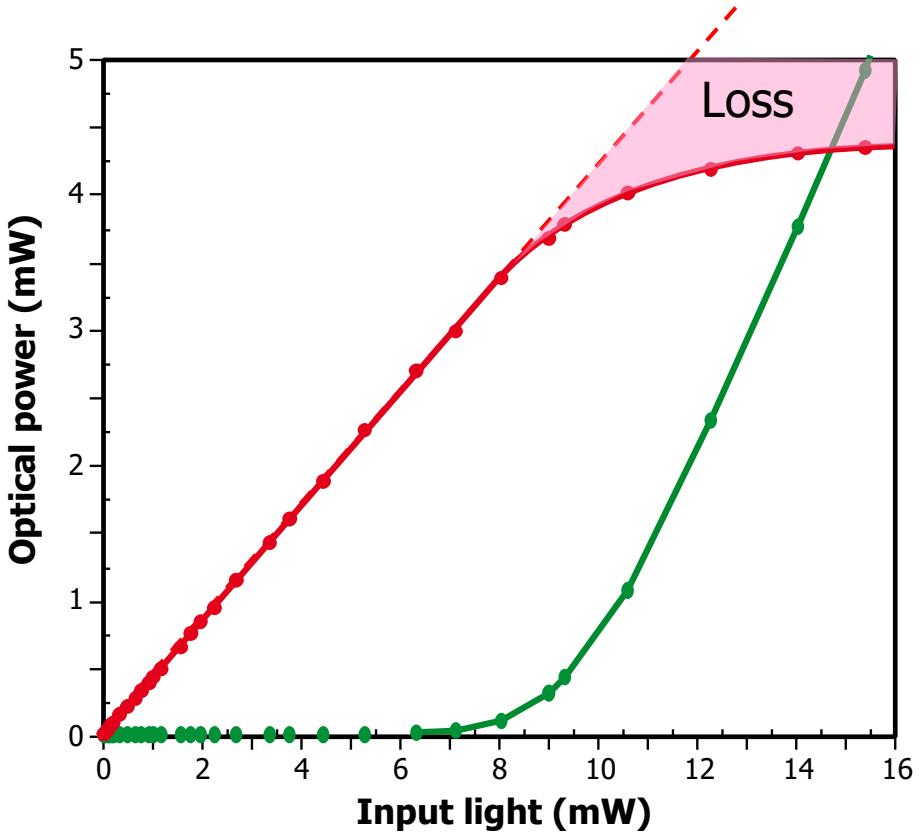


Amplified backscattered light

Brillouin mirror

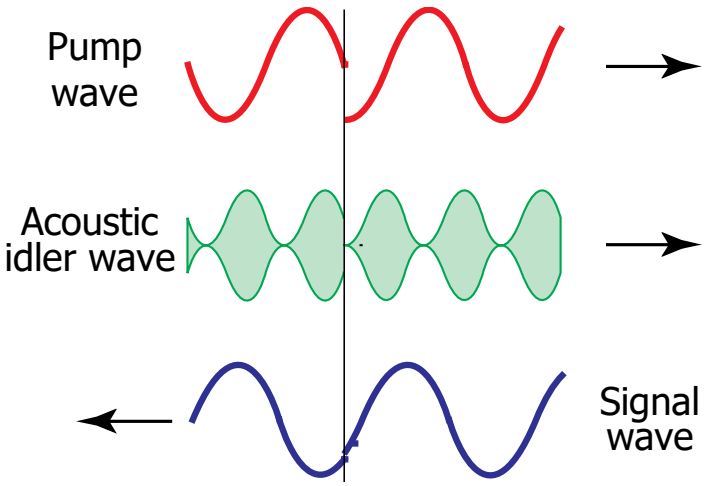
$$P_{crit}^{in} \simeq 16 \frac{A_{eff}}{g_B L_{eff}}$$

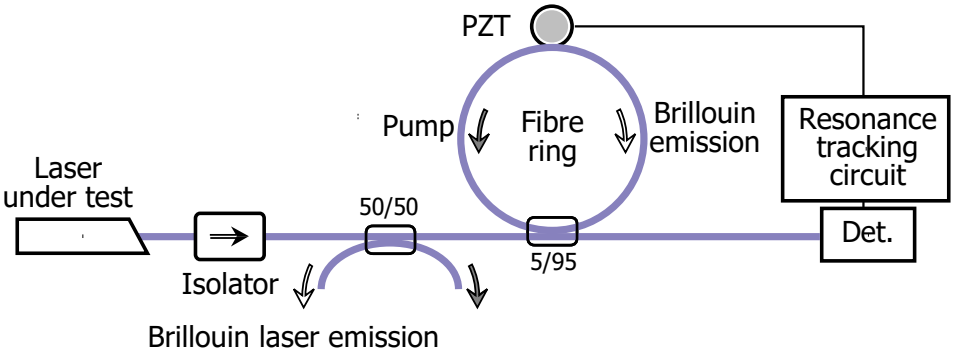
Smith's model



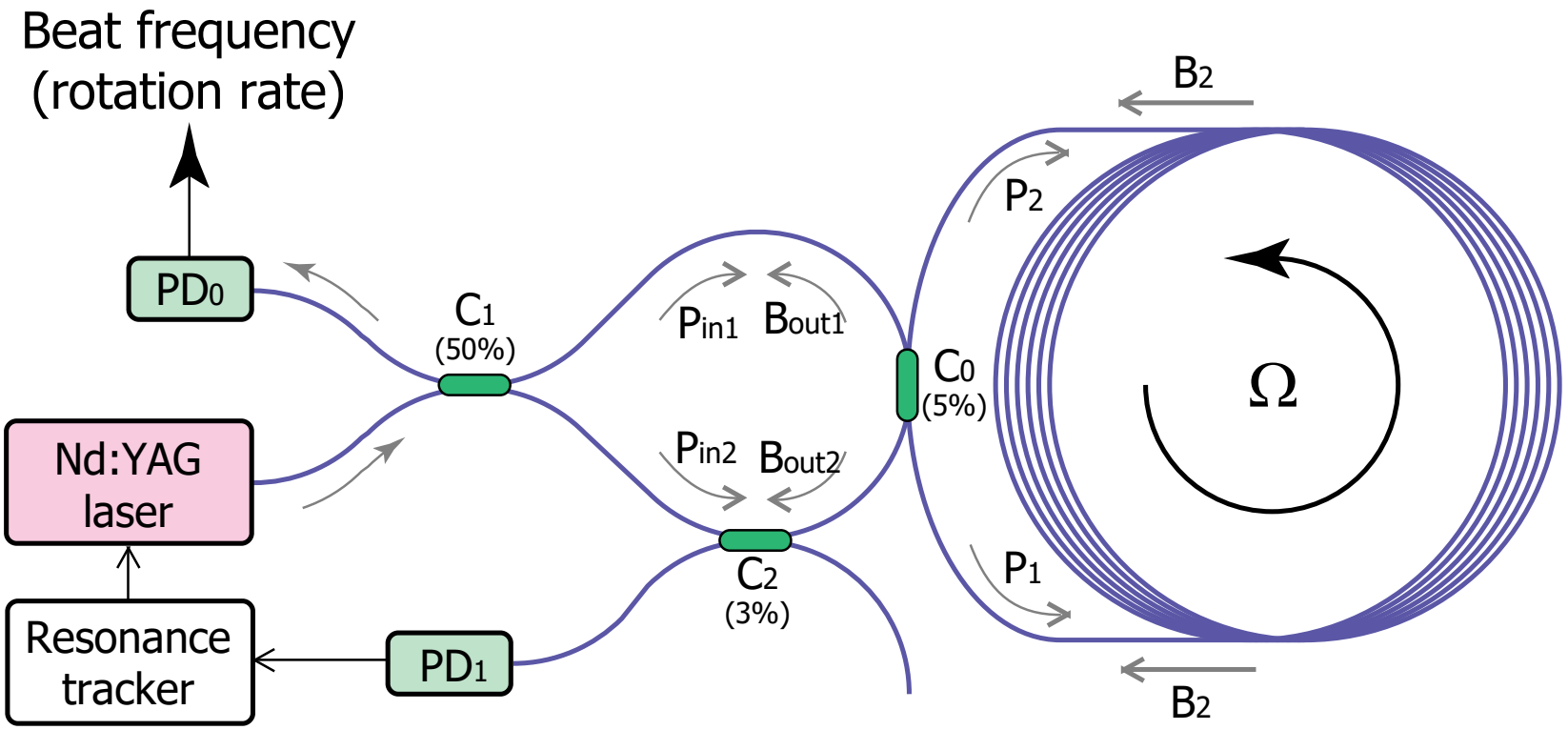
Raman and Brillouin lasers

- Lasers based on stimulated scatterings are easily realized by looping back an amplifier upon itself.
- *Raman*-based fibre lasers can generate light at wavelength well separated from the pump (new wavelength) and even using cascaded emission.
- *Brillouin*-based fibre lasers can show a sub-mW threshold and very high coherence (sub-kHz).





Brillouin Fibre Laser Gyroscope



Brillouin Fibre Laser Gyroscope